Paper Reference(s)

9801/01

Edexcel

Advanced Extension Award

Friday 30 June 2006 – Morning
Time: 3 hours

Materials required for examination
Mathematical Formulae (Green)
Graph paper (ASG2)
Answer Book (AB16)

Items included with question papers
Nil

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initials and signature.

Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. (*a*) For $|y| < 1$, write down the binomial series expansion of $(1 - y)^{-2}$ in ascending powers of y up to and including the term in y^3.

(1)

(*b*) Hence, or otherwise, show that

$$1 + \frac{2x}{1+x} + \frac{3x^2}{(1+x)^2} + \ldots + \frac{rx^{r-1}}{(1+x)^{r-1}} + \ldots$$

can be written in the form $(a + x)^n$. Write down the values of the integers a and n.

(4)

(*c*) Find the set of values of x for which the series in part (*b*) is convergent.

(3)

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2. Given that $(\sin\theta + \cos\theta) \neq 0$, find all the solutions of

$$\frac{2\cos 2\theta(\sin 2\theta - \sqrt{3}\cos 2\theta)}{\sin\theta + \cos\theta} = \sqrt{6}(\sin 2\theta - \sqrt{3}\cos 2\theta)$$

for $0 \leq \theta < 360°$.

(10)

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3. Given that $x > y > 0$,

(*a*) by writing $\log_y x = z$, or otherwise, show that $\log_y x = \dfrac{1}{\log_x y}$.

(2)

(*b*) Given also that $\log_x y = \log_y x$, show that $y = \dfrac{1}{x}$.

(2)

(*c*) Solve the simultaneous equations

$$\log_x y = \log_y x,$$

$$\log_x (x - y) = \log_y (x + y).$$

(7)

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4. The line with equation $y = mx$ is a tangent to the circle C_1 with equation

$$(x + 4)^2 + (y - 7)^2 = 13.$$

(*a*) Show that m satisfies the equation

$$3m^2 + 56m + 36 = 0.$$

(4)

The tangents from the origin O to C_1 touch C_1 at the points A and B.

(*b*) Find the coordinates of the points A and B.

(8)

Another circle C_2 has equation $x^2 + y^2 = 13$. The tangents from the point $(4, -7)$ to C_2 touch it at the points P and Q.

(*c*) Find the coordinates of either the point P or the point Q.

(2)

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5. The lines L_1 and L_2 have vector equations

L_1: $\quad \mathbf{r} = -2\mathbf{i} + 11.5\mathbf{j} + \lambda(3\mathbf{i} - 4\mathbf{j} - \mathbf{k})$,

L_2: $\quad \mathbf{r} = 11.5\mathbf{i} + 3\mathbf{j} + 8.5\mathbf{k} + \mu(7\mathbf{i} + 8\mathbf{j} - 11\mathbf{k})$,

where λ and μ are parameters.

(*a*) Show that L_1 and L_2 do not intersect.

(5)

(*b*) Show that the vector $(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ is perpendicular to both L_1 and L_2.

(2)

The point A lies on L_1, the point B lies on L_2 and AB is perpendicular to both L_1 and L_2.

(*c*) Find the position vector of the point A and the position vector of the point B.

(8)

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6. **Figure 1**

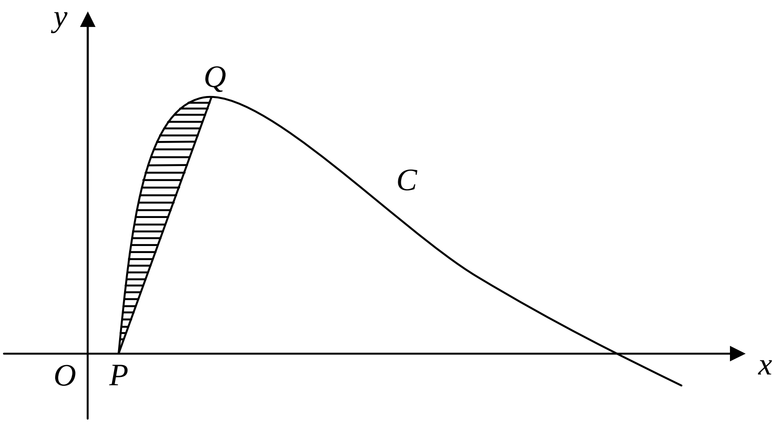

Figure 1 shows a sketch of part of the curve C with equation

$$y = \sin(\ln x), \quad x \geq 1.$$

The point Q, on C, is a maximum.

(a) Show that the point $P(1, 0)$ lies on C.
(1)

(b) Find the coordinates of the point Q.
(5)

(c) Find the area of the shaded region between C and the line PQ.
(9)

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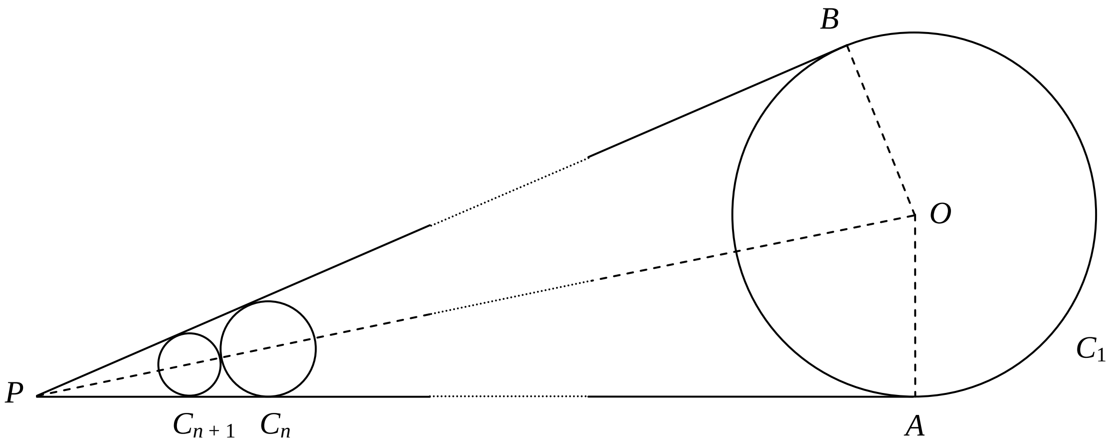
7. **Figure 2**



The circle C_1 has centre O and radius R. The tangents AP and BP to C_1 meet at the point P and angle $APB = 2\alpha$, $0 < \alpha < \frac{\pi}{2}$. A sequence of circles $C_1, C_2, \ldots, C_n, \ldots$ is drawn so that each new circle C_{n+1} touches each of C_n, AP and BP for $n = 1, 2, 3, \ldots$ as shown in Figure 2. The centre of each circle lies on the line OP.

(*a*) Show that the radii of the circles form a geometric sequence with common ratio

$$\frac{1 - \sin\alpha}{1 + \sin\alpha}.$$

(5)

(*b*) Find, in terms of R and α, the total area enclosed by all the circles, simplifying your answer.

(3)

The area inside the quadrilateral $PAOB$, not enclosed by part of C_1 or any of the other circles, is S.

(*c*) Show that

$$S = R^2\left(\alpha + \cot\alpha - \frac{\pi}{4}\operatorname{cosec}\alpha - \frac{\pi}{4}\sin\alpha\right).$$

(5)

(*d*) Show that, as α varies,

$$\frac{dS}{d\alpha} = R^2\cot^2\alpha\left(\frac{\pi}{4}\cos\alpha - 1\right).$$

(4)

(*e*) Find, in terms of R, the least value of S for $\frac{\pi}{6} \le \alpha \le \frac{\pi}{4}$.

(3)

MARKS FOR STYLE, CLARITY AND PRESENTATION: 7 MARKS
TOTAL FOR PAPER: 75 MARKS

END